Swirling-Swepers: Constant Volume Modelling

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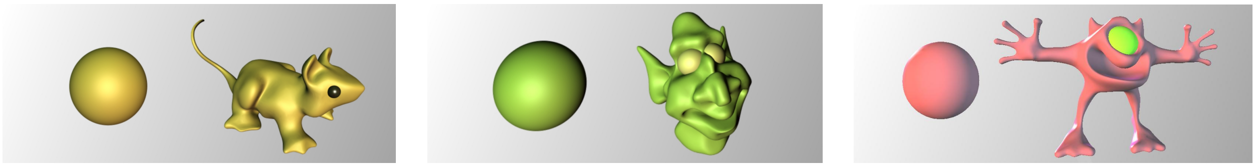

Swirling-sweepers is a new method for modeling shapes while preserving volume. The artist describes a deformation by dragging a point along a path. The method is independent of the geometric representation of the shape. It preserves volume and avoids self-intersections, both local and global. It is capable of unlimited stretching and the deformation can be controlled to affect only a part of the model.

1 Motivation

In a virtual modeling context, there is no material. A challenge for computer graphics is to provide a modeling tool that convinces the artist that there is a material. Volume is one of the most important factors influencing the way an artist models with real materials.

The limitation of existing volume-preserving methods is either that they only apply to a specific type of geometric representation, or they only apply to shapes whose volume can be computed, with the exception of [Decaudin 1996]. His technique does not always preserve volume, and is discontinuous at one point.

2 Principle of Swirling-Sweepers

A Swirling-Sweeper is a new space deformation based on our framework called Sweepers [Angelidis et al. 2004b]. It is a blend of simpler deformations that we call *swirls*. In Figure 1, we show that a swirl is a rotation whose magnitude decreases away from its center, c. We represent the magnitude of rotation by a C^2 monotonic scalar function, ϕ, which vanishes outside a neighborhood of radius λ around c. More formally, a swirl is a rotation matrix raised to the power ϕ

$$f(p) = \exp\left(\phi(||p-c||)\log M\right)p \tag{1}$$

A swirl preserves volume since the determinant of its Jacobian is always equal to 1. There is a convenient way of blending n swirls to produce a more complex deformation

$$f_n(p) = \exp\left(\textstyle\sum_{i=1}^{n}\phi(||p-c_i||)\log M_i\right)p \tag{2}$$

See [Angelidis et al. 2004a] for computing exp and log.

From Swirls to Swirling-Sweepers: By specifying a single translation $\vec{t}$, an artist can input n swirls. As shown in Figure 2, we place n swirl centers, c_i, on the circle of center h, and radius r lying in a plane perpendicular to $\vec{t}$. These points correspond to n consistently-oriented unit tangent vectors $\vec{v}_i$. Each pair, $(c_i, \vec{v}_i)$, together with an angle, θ, define a rotation. Along with radii of neighborhood $\lambda = 2r$, we define n swirls. The radius of the circle, r, is left to the user to choose. The following value for θ will transform h exactly into $h + \vec{t}$, and preserves volume for sufficiently small $\vec{t}$:

$$\theta = \frac{2||\vec{t}\,||}{nr} \tag{3}$$

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Preserving coherency and volume: To preserve coherency and volume, it is necessary to subdivide input vector $\vec{t}$ into a series of smaller vectors. We use $s = \max(1, \lceil -4||\vec{t}\,||/r\rceil)$ sub-vectors. This decomposition is shown in Figure 3.

Achieving Real-Time: We have a closed-form for the logarithm of a rotation matrix and also for computing $(\exp N)p$, when N is the logarithm of an unknown rotation matrix. These formulas, together with a more complete discussion can be found in [Angelidis et al. 2004a]. They save otherwise expensive numerical approximations.

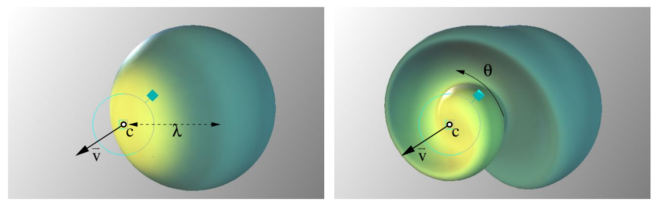

Figure 1: Swirl of center c, rotation angle θ around $\vec{v}$.

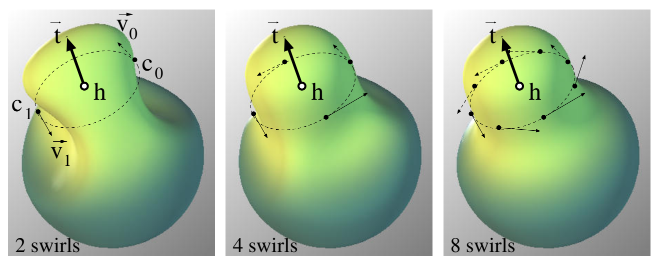

Figure 2: n swirls arranged in a ring creates more complex deformations. There are no visible artifacts with 8 swirls.

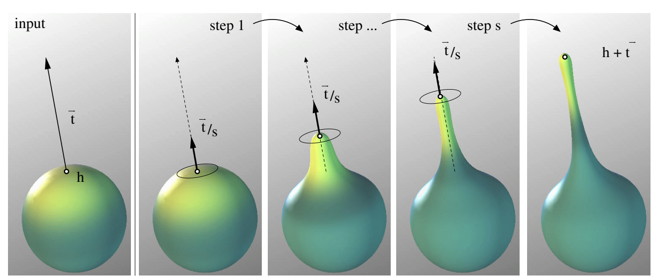

Figure 3: Volume preservation is obtained by composing rings of swirls. The selected point is precisely controlled.

References

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Decaudin, P. 1996. Geometric deformation by merging a 3d object with a simple shape. In *Graphics Interface*, 55–60.